No. 10

Computations of national forest timber survey data.
Curve fitting and volume table checks and revisions.
February 1935. L. I. Barrett.

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Computations of National Forest Timber Survey Data

Curve Fitting and Volume Table Checks and Revisions

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no. 10

The purpose of this technical note is to outline procedures particularly adapted to the computation of the present timber surveys on National Forests. The responsibility for much of this work has of necessity been placed upon Junior Foresters, many of whom are for the first time faced with the assembling and computation of large quantities of management plan data. It is to these men that the following discussions are primarily directed. In this and forthcoming Technical Notes the various procedures dealt with were chosen as important after conferences with forest officers engaged in the direction of timber surveys and management plan work.

The immediate need is for volume tables which can be used with confidence on the different forests. Because the checking and revision of volume tables involves considerable use of free hand curves, the first section of this note will be devoted to practical suggestions for the proper fitting of such curves. The second section will outline in detail the various steps for checking and, if necessary, revising existing regional tables for local use on the national forests.

1 - Curve Fitting

The curve of merchantable height over d.b.h. is probably the first curve with which the timber survey computer comes in contact. For this reason such a curve will be used as an illustration of curve fitting. Under the present system for each species a merchantable height over d.b.h. curve is drawn for each of the 3 site classes. In preparation for plotting on cross-section paper, computations of sample trees should be summarized as in Table 1. The average merchantable height (column 3), of each inch class should then be plotted over the actual average d.b.h. of the diameter class as shown in column 2. Plotted points representing an average must always be accompanied on the cross section paper by the number of items (column 4 of Table 1) upon which the average is based (See the small figures near each point on Fig. 1) After plotting the points, draw a smooth curve through them by inspection. This curve is probably not an accurate expression of the data, but furnishes a basis to work from in revising the position and shape of the curve. This revision is accomplished by the following steps:

1. Count the number of small squares between the first plotted point and the curve. The counting must be done vertically. (For the first point on Fig. 1 the number of squares is 1.5).

2. Multiply the number of squares by the number of items (in this case number of trees) which were used to determine the average of the particular class represented by the point. (On Fig. 1 1.5 x 3 = 4.5)

3. If the point is above the curve, record the product derived in step 2 under a plus sign on the sheet (See Fig. 1) If the point is be-

FIG. 1

Merch. Ht. - feet

70
60
50
40
30

10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42

D.B.H. - inches

First Fit	
+	−
4.5	64.0
82.0	64.0
40.0	16.0
33.0	73.5
35.0	45.0
8.0	30.0
4.0	18.0
	39.0
	11.0
	32.0
206.5	392.5

Final Fit	
+	−
4.5	64.0
105.0	24.0
32.0	3.0
48.0	56.0
50.0	36.0
18.0	27.5
10.0	15.0
	31.5
	5.0
	29.0
287.5	293.0

----- First Fit

——— Final Fit

Table I

Site I - Yellow Poplar

Merchantable Heights.

1 DBH Class	2 Average DBH	3 Av. Merchant- able Height	4 Number of trees
10	10.4	41	3
12	12.0	45	16
14	13.9	55	21
16	15.8	55	16
18	17.9	57	4
20	19.4	65	4
22	21.8	56	7
24	24.0	55	3
26	25.5	48	1
28	28.2	67	6
30	29.9	56	1
32	31.5	59	3
34	33.9	70	5
36	35.7	68	4
38	37.6	65	2
40	39.0	52	1
42	41.9	70	2

low the curve, record it under a minus sign.

4. Proceed in this manner with each plotted point, recording the products derived from points above the curve under a plus sign, and those below the curve under a minus sign.

5. When all products have been worked out, compare the sum of the plus products with the sum of the minus products. If the curve is an adequate expression of the data, these sums should be approximately equal.

In Figure 1 the broken line represents the first curve drawn through the plotted points. It is obvious from the sums of the plus and minus products shown in Figure 1 under the heading "First Fit" that the minus products exceed the plus products by 186 (392.5 - 206.5) This indicates that there is too much weight below the curve and that it must be lowered to give a better expression of the data. The curve was then lowered as indicated by the solid line and its shape changed slightly and new products computed. Under the heading "Final Fit" of Fig. 1 it is apparent that the sum of the plus products is approximately equal to the sum of the minus products. The difference between these two values should not exceed 1 percent of their sum if the curve is properly fitted. In this case the fit is not highly satisfactory since the difference between the sums of the plus and minus products is 5.5 (293.0 - 287.5) while 1% of their total is 5.8 (1% of 293.0 + 287.5)

A further and equally important test of the adequacy of the final curve is obtained by comparing the sums of all products regardless of their algebraic sign, with a similar sum for other trial fits. For example, the sum of the products of the first fit is 599 (206.5 + 392.5) while the corresponding sum of the final fit is 580.5 (287.5 + 293.0) a lower figure than for the first fit. Since these sums are a measure of the total scatter of the data around the curve, the lower figure (580.5) shows a closer correlation between the final curve and the plotted points than did the first curve. The mere balancing of the plus and minus products is not then, the final test of any curve, since a very inadequate curve may show these products to balance. For instance, in Figure 1 a straight horizontal line could have been fitted to the plotted points in such a position that the sum of the plus and the sum of the minus products were equal. However, the figure obtained by adding these two sums would greatly exceed the similar figure for the final fit (580.5). Therefore, a curve is not a correct expression of the data until the sum of the plus and minus products balance and until the total of these plus and minus products is the lowest possible figure. However, this last statement must not be interpreted as a recommendation for S-shaped curves or curves of a wave like character in which several crests and troughs are shown. Past experience has proved that the simple correlations such as presented here are most perfectly expressed by smooth curves without irregularities. Plotted points indicating S-shaped or other irregular curves are usually the result of inadequate basic data.

II. Volume Table Checks and Revisions

The revision of regional volume tables for local use consists basically of comparing the volumes of locally cut trees with the volumes of these same trees as derived from the tables for the particular species in question. In case the table does not fit the local data, the scaled volumes of the locally cut trees are used to correct the table. The basic

TABLE NO.II
Yellow Poplar

Correction of Volume Tables for Local Use

1	2	3	4	5	6	7	1	2	3	4	5	6	7
			Volume in bd.ft.							Volume in bd.ft.			
Tree No.	DBH In.	No. logs	from table	local trees	Diff.	Diff. in %	Tree No.	DBH In.	No. logs	from table	local trees	Diff.	Diff. in %
1	10.0	2	50	60	10	20.0	38	14.7	3 7/8	255	360	105	41.2
2	10.4	2 1/2	71	70	1	1.4	39	14.8	3 1/4	210	160	50	23.8
3	10.8	3 1/4	106	90	16	15.1	40	14.9	4 1/8	280	180	100	35.7
4	11.0	3	100	110	10	10.0	41	15.0	3 3/4	255	240	15	5.9
5	11.0	2 3/4	90	70	20	22.2	42	15.0	2 5/8	161	160	1	0.6
6	11.0	3	100	90	10	10.0	43	15.1	3 1/8	238	190	48	20.2
7	11.2	2 7/8	99	80	19	19.2	44	15.2	4 1/4	300	240	60	20.0
8	11.7	2	67	70	3	4.5	45	15.4	3 5/8	258	200	58	22.5
9	11.8	3 1/4	130	180	50	38.5	46	15.4	3 3/8	236	200	36	15.2
10	11.9	2	69	70	1	1.4	47	15.6	3 1/4	274	240	34	12.4
11	12.0	2 1/2	95	80	15	15.8	48	15.6	4 3/8	322	260	62	19.2
12	12.1	2 5/8	103	100	3	2.9	49	15.6	2 7/8	198	220	22	11.1
13	12.2	3 1/4	140	110	30	21.4	50	15.7	2 3/8	152	120	32	21.0
14	12.4	2	74	100	26	35.1	51	15.9	3 1/4	240	270	30	12.5
15.	12.4	3 5/8	169	140	29	17.2	52	16.6	3 7/8	322	230	92	28.5
16	12.6	3 7/8	190	140	50	26.3	53	16.6	2 3/8	169	170	1	0.6
17	12.6	3 1/2	166	140	26	15.6	54	16.6	4 3/8	361	340	21	5.8
18	12.7	2 1/4	92	100	8	8.7	55	16.7	3 3/4	313	260	53	16.9
19	12.8	2 1/4	94	75	19	20.2	56	16.7	3	238	180	58	24.4
20	13.0	3 1/2	178	130	48	26.9	57	17.1	4 1/4	373	290	83	22.2
21	13.0	3 1/2	178	140	38	21.3	58	17.1	3	248	270	22	8.9
22	13.0	3 5/8	186	150	36	19.3	59	18.6	3 3/4	385	310	75	19.5
23	13.2	4 1/8	224	210	14	6.2	60	18.7	3 1/4	324	260	64	19.7
24	13.2	3	150	120	30	20.0							
25	13.3	3 1/4	169	120	49	29.0	61	19.0	4 3/4	514	240	274	53.3
26	13.4	2 5/8	128	120	8	6.2	62	19.1	4 1/4	466	280	186	40.0
27	13.6	3 5/8	202	180	22	10.9							
28	13.6	3 7/8	220	190	30	13.6	63	21.3	3	369	330	39	10.6
29	13.7	3 3/8	188	150	38	20.2				12857	10845		1170.1
30	13.7	3	162	110	52	32.1				-10845			
31	13.8	2 1/4	107	130	23	21.5				2012			
32	14.0	2 3/4	150	110	40	26.6							
33	14.2	3	175	140	35	20.0							
34	14.4	3 1/8	189	180	9	4.8							
35	14.4	4	254	220	34	13.4							
36	14.6	4 1/8	270	210	60	22.2							
37	14.6	4	261	160	101	38.7							

*$\frac{201200}{12857}$ = 15.6% - Aggregate Difference in percent

Table is 15.6% high in aggregate.

$\frac{1170.1}{63}$ = ± 18.6% - Average individual deviation in %

* Two zeros are added for computation of percentage.

	2 log	3 log
10 in.	50	80
12 in.	70	120

The tree for which the volume table value is to be computed has a d.b.h. of 10.4 and contains $2\frac{1}{2}$ logs. The next step is to determine the volume table value of 2 and 3 log trees which are 10.4 inches in diameter. There is a spread of 20 tenth inches between 10.0 and 12.0 so that a 2 log tree 10.4 inches in d.b.h. will have a volume equal to the volume of a 10.0 inch, 2 log tree plus $\frac{4}{20}$ of the difference in volume between a 10.0 inch 2 log tree and a 12.0 inch 2 log tree. In this case the computation is $\frac{4}{20}$ x (70 - 50)=4. A 10.4 inch 2 log tree then has an estimated volume of 50 + 4 or 54 board feet. A similar computation for 3 log trees shows the volume of a 10.4 inch, 3 log tree to be 88 feet, $\frac{4}{20}$ x (120 - 80) + 80 = 88.

The computation then appears as follows:

	2 log	$2\frac{1}{2}$ log	3 log
10	50		80
10.4	54		88
12	70		120

Continuing the computation, a 10.4 inch, $2\frac{1}{2}$ log tree then has an estimated volume of 54 + 1/2 (88 - 54) = 71 board feet which is the volume recorded in column 4.

This interpolative method appears laborious, but failure to follow it will result in errors in columns 6 and 7 which are unfair to the table and which will result in showing larger measures of error (see later discussion of Aggregate Difference and Average Individual Deviation) than actually exist in the table.

3. Obtain totals for columns 4 and 5. The difference between the two totals expressed as a percentage indicates the aggregate error of the table and whether it is high or low, a plus sign before the percentage indicating the table is high, and vice versa. If the totals of the volumes estimated from the volume table (col. 4) exceed the total volume of local trees (col. 5) the volume table will give results in excess of actual values and vice versa. Note the computation of Aggregate Difference at the end of Table II. The Aggregate Difference measures the total amount of error in the volume table, or stated differently, it measures the amount of error which will result from use of the table in computing volumes based in a timber cruise provided the diameters measured in the cruise contain the entire range of diameters present in the volume table.

Another and equally important measure of accuracy must be made. This is accomplished by the following steps:

4. With column 4 filled in, obtain the individual differences of the values in columns 4 and 5 and enter in column 6. Column 6 then merely lists the differences in board foot volume between actual local scale and computed volume table values for trees of the same dimensions.

5. Using columns 4 and 6, compute the individual percentages of error found in the table and enter these percentages in column 7.

6. Total the values in column 7 and divide by the number of trees

used in the test to obtain the Average Individual Deviation.

As already pointed out, the aggregate difference of +15.6% in this particular case indicates that the table gives much higher values for the local trees than actually exist. It is entirely possible for the aggregate difference to be only 1% and for the table to still be quite inaccurate. This is due to the fact that a table may give very high values for small trees and low values for large trees which compensate to give a very low aggregate difference. If such a table is applied to pole stands where the majority of the saw log timber is small, the table may be considerably in error. This is also true where only a few trees of a given species are present in a stand. It is under such conditions that the Average Individual Deviation comes into the picture. This measure of accuracy indicates the possible amount of error which may be expected in case volumes are computed for only a limited number of trees or diameter classes.

In general a volume table should have an aggregate difference of not over 2% and the average individual deviation should not be greatly in excess of ± 15%.

After completing step 3, if the Aggregate Difference is in excess of 2% showing that the volume table needs correction, the computation of the Average Individual Deviation (Step 4, 5 and 6) may be postponed until later, and step 7 begun immediately. In cases where the Aggregate Difference is high the Average Individual Deviation means little or nothing and time will be saved by postponing its computation until the volume table is revised. In the case presented in Table II it is obvious that the volume table needs correction.

The next steps are as follows:

7. For each d.b.h. class obtain totals for columns 4 and 5 (Table II) and enter these totals in columns 3 and 4 of Table III. Also determine the average d.b.h. of each diameter class and record in column 2.

Table III
Yellow Poplar

1 DBH Class	2 Average DBH	3 Total Volume from table	4 Total vol. scale local trees	5 Diff. in bd. ft.	6 Diff. in % of table volume	7 Number of Trees
10	10.4	227	220	+7	+3.1	3
12	12.0	1778	1655	+123	+6.9	16
14	13.9	4136	3470	+666	+16.1	21
16	15.8	3876	3520	+356	+ 9.2	16
18	17.9	1330	1130	+200	+15.0	4
20	19.0	980	520	+460	+47.0	2
22	21.3	369	330	+ 39	+10.6	1
						63

FIG. 2

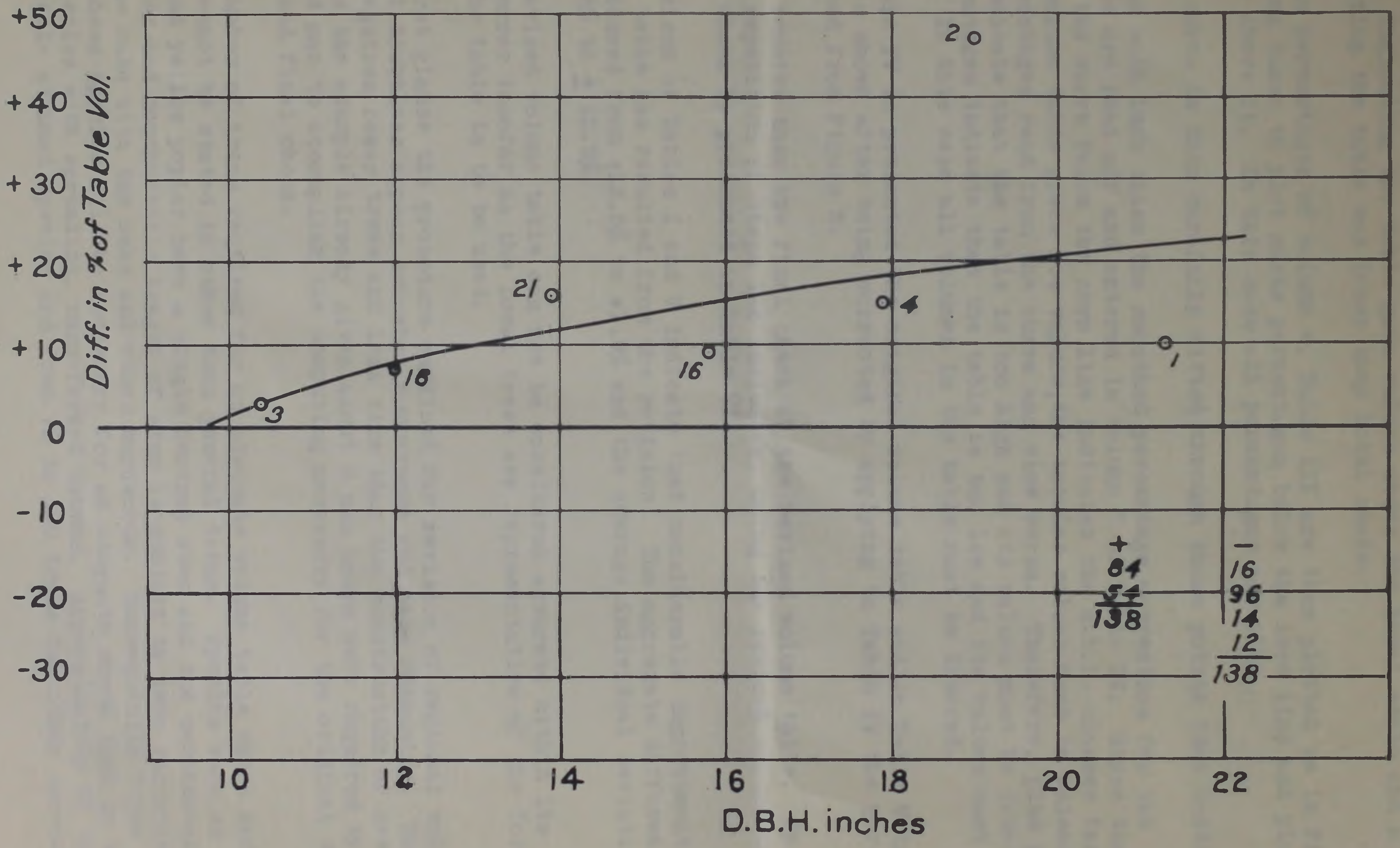

+50
+40
+30
+20
+10
0
-10
-20
-30
Diff. in % of Table Vol.
20
21
18
16
4
1
3
+
84
54
138
−
16
96
14
12
138
10
12
14
16
18
20
22
D.B.H. inches

Table IV
Yellow Poplar
Original Volume Table
to be checked

1	2	3	4	5	6	7
		Number of 16 foot logs				Lower
DBH Class	1	2	3	4	5	Volume by
		Volume in board feet				1.5%
10	30	50	80	110		
12	40	70	120	180	240	8.0
14	60	90	170	240	310	12.0
16	80	120	220	310	380	15.5
18	110	150	270	390	470	18.5
20	160	180	330	480	610	21.0
22		220	390	590	760	23.0

Example:
From Table IV the volume of a 22 inch, 4 log tree = 590 board feet.
From Table IV, col. 7 this volume must be lowered by 23%.
0.23 x 590 = 135.7 bd. ft. 590 - 136 = 454, or 450 Dec. C.
This volume (450) is then entered in its proper place in Table V.

Table V

Corrected volume table

DBH Class	Number of 16 foot logs 1	2	3	4	5
	Volume - board feet				
10	30	50	80	110	
12	40	60	110	170	220
14	50	80	150	210	270
16	70	100	190	260	320
18	90	120	220	320	380
20	130	140	260	380	480
22		170	300	450	590

Aggregate Difference = + 1.9%
Average Individual Deviation = ± 15.2%
} measures of error of revised table.

should be the minimum. The sizes of the trees measured locally should be well distributed over the range of sizes included in the volume table to be checked.

A further criterion of whether sufficient trees have been measured for an adequate check can be obtained by inspection of the plotted points of Fig. 2 and the magnitude of the final average individual deviation.

After the volume table has been checked and corrected for local conditions there remains the problem of revising it so that volumes for trees of a given d.b.h. and site class can be determined. For this purpose the volume table can be transferred into graphic form as shown by the broken lines of Fig. 3. In doing this it must be remembered that the volume for a two log tree must be plotted opposite a merchantable height on the vertical ordinate of the gruph paper equal to two 16 foot sections plus a trimming allowance 0.3 feet per log. Volumes for the various log heights must be plotted opposite their corresponding merchantable heights. Such heights are:

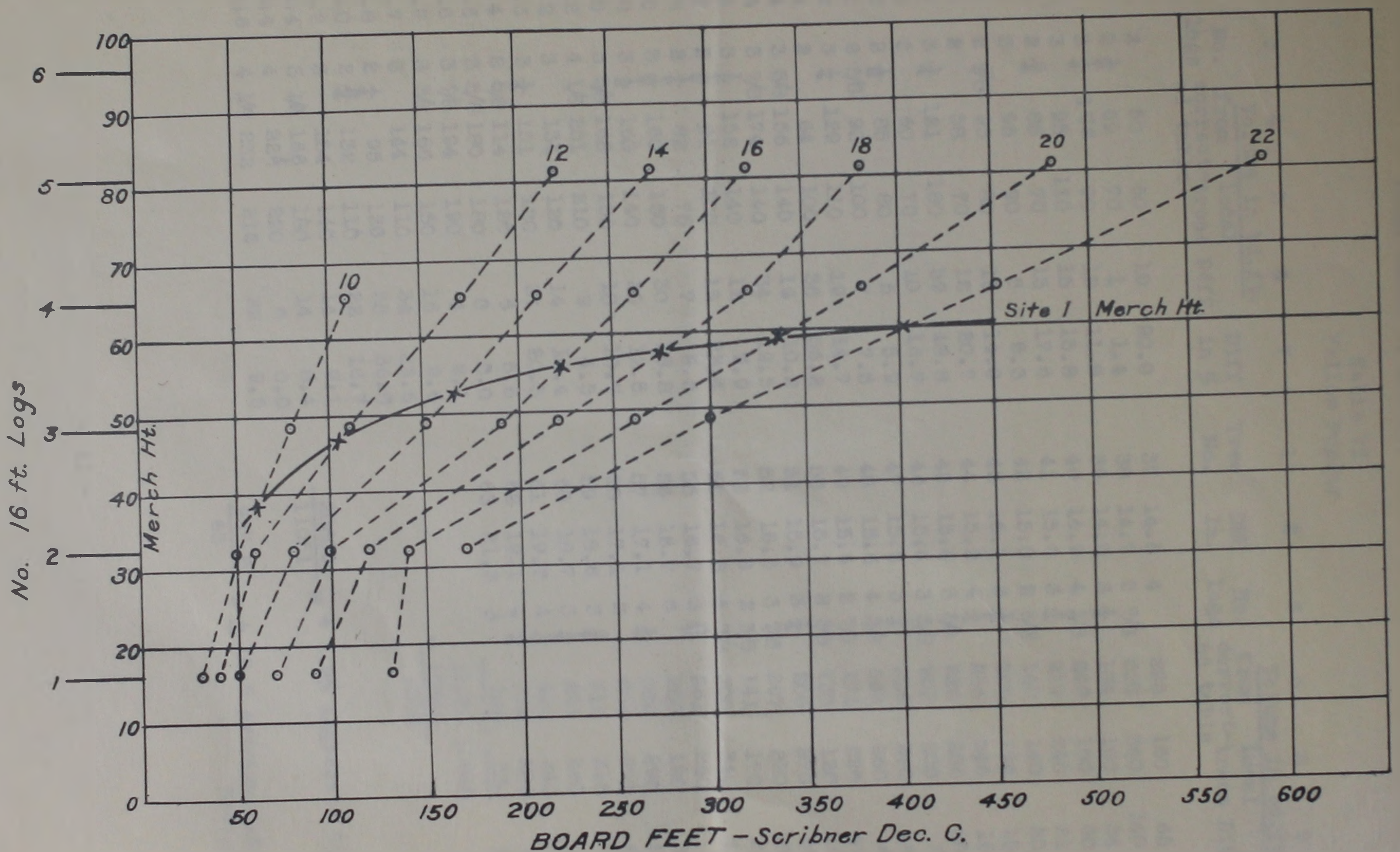
No. logs	Merchantable length.
1	16.3
2	32.6
3	48.9
4	65.2
5	81.5
6	97.8

For each site class a curve of merchantable height over d.b.h. can then be constructed for each species from sample tree data as shown in Fig. 1. The smoothed values from this curve can then be transferred to Fig. 3, plotting on each d.b.h. class curve the corresponding merchantable length of stem. A new volume table showing tree volumes by d.b.h. and site class can then be read from Fig. 3.

It is possible of course to construct a base table such as Table V for each site class. This requires only that the local trees used in the check be segregated by site class. All trees measured on Site I would then be used to revise the original table to get a new Site I volume table and likewise with Sites II and III. However, it is doubtful if a sufficient number of trees in all three sites can be collected so that the three separate site class volume tables can be built. From 2 to 3 months work on 2 national forests of Region 8 have shown that the majority of the trees measured on logging jobs come from Site II and that the number of trees from Sites I and III are insufficient for individual volume table checks. Consequently, for the present job it is recommended that all trees of a single species be measured on local logging jobs and thrown together regardless of site for the purpose of ehcking and revising regional volume tables for local use. This revised volume table can then be used as the basis for plotting such data as Fig. 3. One of the most important effects of site on tree volume will then be provided for by the merchantable height curves superimposed on such curves as Figure 3. However, a site class should be assigned to each tree scaled for the purpose of checking volume tables. In the course of time enough data will accumulate so that an investigation can be made of the effect of site on the volumes of trees of the same dimensions, and if found to be significant new base tables by site classes can be made for future use.

Volume tables derived from the foregoing procedure give gross volumes when applied to any given area of forest land. These gross volumes must be

FIG. 3


100
90
80
70
60
50
40
30
20
10
0
6
5
4
3
2
1
Merch Ht.
No. 16 ft. Logs
10
12
14
16
18
20
22
Site I Merch Ht.
50
100
150
200
250
300
350
400
450
500
550
600
BOARD FEET – Scribner Dec. C.

Final check of Revised Volume Table

Table VI
Yellow Poplar

1	2	3	4	5	6	7	1	2	3	4	5	6	7
			Volume in bd.ft.							Volume in bd.ft.			
Tree No.	DBH In.	No. logs	from correct-ed table	local trees	Diff.	Diff in %	Tree No.	DBH In.	No. logs	from correct-ed table	local trees	Diff.	Diff. in %
1	10.0	2	50	60	10	20.0	37	14.6	4	225	160	65	28.9
2	10.4	2 1/2	69	70	1	1.4	38	14.7	3 7/8	220	360	140	63.6
3	10.8	3 1/4	102	90	12	11.8	39	14.8	3 1/4	184	160	24	13.0
4	11.0	3	95	110	15	15.8	40	14.9	4 1/8	240	180	60	25.0
5	11.0	2 3/4	85	70	15	17.6	41	15.0	3 3/4	219	240	21	9.6
6	11.0	3	95	90	5	5.3	42	15.0	2 5/8	140	160	20	14.3
7	11.2	2 7/8	93	80	13	14.0	43	15.1	3 1/2	205	190	15	7.3
8	11.7	2	58	70	12	20.7	44	15.2	4 1/4	255	240	15	5.9
9	11.8	3 1/4	121	180	59	48.8	45	15.4	3 5/8	220	200	20	9.1
10	11.9	2	60	70	10	16.7	46	15.4	3 3/8	203	200	3	1.5
11	12.0	2 1/2	85	80	5	5.9	47	15.6	3 3/4	226	240	14	6.2
12	12.1	2 5/8	93	100	7	7.5	48	15.6	4 3/8	266	260	6	2.3
13	12.2	3 1/4	129	110	19	14.7	49	15.6	2 7/8	171	220	49	28.6
14	12.4	2	64	100	36	56.2	50	15.7	2 3/8	130	120	10	7.7
15	12.4	3 5/8	156	140	16	10.2	51	15.9	3 1/4	205	270	65	31.7
16.	12.6	3 7/8	174	140	34	19.5	52	16.6	3 7/8	267	230	37	13.9
17	12.6	3 1/2	152	140	12	7.9	53	16.6	2 3/8	141	170	29	20.6
18	12.7	2 1/2	81	100	19	23.4	54	16.6	4 3/8	300	340	40	13.3
19	12.8	2 1/4	82	75	7	8.5	55	16.7	3 3/4	261	260	1	.4
20	13.0	3 1/2	160	130	30	18.8	56	16.7	3	200	180	20	10.0
21	13.0	3 1/2	160	140	20	12.5	57	17.1	4 1/4	309	290	19	6.2
22	13.0	3 5/8	168	150	18	10.7	58	17.1	3	206	270	64	31.0
23	13.2	4 1/8	201	210	9	4.5	59	18.6	3 3/4	312	310	2	.6
24	13.2	3	134	120	14	10.4	60	18.7	3 1/4	261	260	1	.4
25	13.3	3 1/4	151	120	31	20.5	61	19.0	4 1/4	410	240	170	41.5
26	13.4	2 5/8	114	120	6	5.6	62	19.1	4 1/4	374	280	94	25.1
27	13.6	3 5/8	180	180	0	0.0	63	21.3	3	286	330	44	15.4
28	13.6	3 7/8	194	190	4	2.1							
29	13.7	3 3/8	166	150	16	9.6				11051	10845		957.8
30	13.7	3	144	110	34	23.6				- 10845			
31	13.8	2 1/4	95	130	35	36.8				-206			
32	14.0	2 3/4	132	110	22	16.7							
33	14.2	3	154	140	14	9.1							
34	14.4	3 1/8	166	180	14	8.4							
35	14.4	4	220	220	0	0.0							
36	14.6	4 1/8	232	210	22	9.5							

$\frac{20600}{11051}$ = + 1.9% Aggregate Difference

$\frac{957.8}{63}$ = ± 15.2 Average Individual Deviation

corrected for cull percent.

Cull per cent for each d.b.h. class by species can be computed from the cull found in the trees measured locally.for volume table checking.

The steps in such a method are as follows:

1. For a single species obtain total gross scale in each d.b.h. class. This has already been done in column 4 of Table III.

2. Obtain for each d.b.h. class the total volume scaled as defect.

3. From the data obtained in steps 1 and 2 determine for each d.b.h. class the percentage of gross volume represented by defect.

4. Plot these percentages over the average d.b.h. of each diameter class, fit a curve and read from the curve the average percentage of cull for each diameter class.

Diameter class gross volumes for any compartment or other area may then be reduced by the average percentages obtained from the curve of step 4.